

國立中正大學

108 學年度碩士班招生考試

試題

[第 1 節]

系所組別	數學系統計科學
科目名稱	基礎數學

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

This exam has 6 questions, for a total of 100 points.

1. Let $f_1(x) = \frac{1}{\sin x} - \frac{1}{x}$ and $f_2(x) = (1 - \frac{1}{x})^x, x \in \mathbb{R}$. Find

- (a) (5 points) $\lim_{x \rightarrow 0} f_1(x)$.
(b) (5 points) $\lim_{x \rightarrow \infty} f_2(x)$.

2. Prove that

- (a) (10 points) $e^x > 1 + x + \frac{x^2}{2}$ for $x > 0$,
(b) (10 points) $e^x < 1 + x + \frac{x^2}{2}$ for $x < 0$.

3. (a) (10 points) Find

$$\int_0^1 \frac{x^2}{\sqrt{1-x^6}} dx.$$

- (b) (10 points) Let $g(x) = e^{-\frac{x^2}{2}}, x \in \mathbb{R}$. Approximate the value of

$$\int_{-1}^2 g(x) dx.$$

(Hint: if Z is a standard normal random variable, $\Pr(|Z| \leq 1) \approx .68$ and $\Pr(|Z| \leq 2) \approx .95$.)

4. Let A, B , and C be $n \times n$ real matrices, $n > 1$. Determine whether each of the following statements is true or false. If a statement is true, prove it. If a statement is false, give a counterexample.

- (a) (5 points) If $AC = BC$, then $A = B$.
(b) (5 points) If A and B are invertible, then AB is invertible.
(c) (5 points) If A is invertible, then the system of linear equation $AX = b$ has a unique solution.

5. (20 points) Find two variables u and v and two values λ_1 and λ_2 such that

$$5x^2 - 2xy + 5y^2 = 4$$

can be rewritten as

$$\lambda_1 u^2 + \lambda_2 v^2 = 4,$$

where $u = ax + by$ and $v = cx + dy$ with $a, b, c, d \in \mathbb{R}$.

6. Let \mathbf{a} be an $n \times 1$ nonzero vector, where $n > 1$ and \mathbf{a}^T be the transpose of \mathbf{a} . Denote $\mathbf{A} = \mathbf{a}\mathbf{a}^T$. Find

- (a) (10 points) the eigenvalues of \mathbf{A} ,
(b) (5 points) the determinant of \mathbf{A} .