(10%) 1. Calculate the following formula:

(5%) (a) $\sin(2\cos^{-1}(\frac{4}{5}))$.

(5%) (b) $\cot^{-1}(\tan(\frac{4\pi}{3}))$.

(10%) 2. Find the following limit:

(5%) (a) $\lim_{x\to 0}(\frac{1}{x^2} - \frac{\cot(x)}{x})$.

(5%) (b) $\lim_{x\to 1^+}(\ln(x))^{1-x}$.

(10%) 3. Find a and b such that $\lim_{x\to 0}\frac{ax+\sin(bx)}{x^3} = 36$.

(10%) 4. Let $f(x) = x^4 + 5x^3 + 3x^2 + x + 1$. Express $f(x)$ as a polynomial of $(x-1)$.

(15%) 5. Evaluate the following integral:

(5%) (a) $\int_0^1 x\cos(x)dx$.

(5%) (b) $\int_0^{\ln(2)} e^x \ln(1+e^x)dx$.

(5%) (c) $\int_{-1}^{2} \frac{1}{9x^2+6x+5}dx$.

(10%) 6. Evaluate the following series:

(5%) (a) $\sum_{k=1}^{\infty}\frac{3^k+5^k}{3^k 5^k}$.

(5%) (b) $\sum_{k=1}^{\infty}\frac{x^2}{(1+x^2)^k}$.

(5%) 7. Show that A and A^T have the same characteristic polynomial.

(10%) 8. Let $A=\begin{bmatrix}1 & 2\\ 4 & 3\end{bmatrix}$. Find a non-singular matrix, P, such that

$$P^{-1}AP=\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}.$$

(10%) 9. Suppose that $T: R^2 \to R^2$ is a linear transformation such that

$$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right)=\begin{bmatrix}2\\3\end{bmatrix} \text{ and } T\left(\begin{bmatrix}1\\-1\end{bmatrix}\right)=\begin{bmatrix}4\\-1\end{bmatrix}.$$

(3%) (a) Find $T\left(\begin{bmatrix}3\\3\end{bmatrix}\right)$.

(7%) (b) Find $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)$ and $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$, and use the results to determine

$T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right)$.

(10%) 10. Let u and v be eigenvectors of a symmetric matrix that correspond to distinct eigenvalues. Show that u and v are orthogonal.