國立中正大學
111 學年度碩士班招生考試
試 題
[第2節]

科目名稱	機率與統計
系所組別	數學系統計科學

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

(20%) 1. A random variable X has a probability density function given by $f(x) = \frac{1}{2}e^{-|x|}$, $x \in \mathbb{R}$. Find the moment generating function of X.

(20%) 2. Let $\phi(z)$, $z \in \mathbb{R}$, be the probability density function of a standard normal random variable. For $x, y \in \mathbb{R}$, the joint probability density function of random variables X and Y is defined by

$$f_{X,Y}(x,y) = \begin{cases} (1+\varepsilon)h(x,y), & \text{if } xy \geq 0; \\ (1-\varepsilon)h(x,y), & \text{if } xy < 0, \end{cases}$$

where $0 < \varepsilon < 1$ is a constant and $h(x,y) = \phi(x)\phi(y)$. Find the marginal probability density function of X.

(20%) 3. Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables with common continuous distribution function F. For $n \in \mathbb{N}$, define

$$Y_n = \begin{cases} X_1, & \text{if } n = 1; \\ \max\{X_1, X_2, \ldots, X_n\}, & \text{if } n \geq 2. \end{cases}$$

Find the limiting distribution of $n\big(1 - F(Y_n)\big)$.

(20%) 4. Let $X_1, X_2, \ldots, X_n$ be a random sample from a $\mathrm{N}(\mu, \sigma^2)$ population with $-\infty < \mu < \infty$ and $0 < \sigma < \infty$. Define

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \text{ and } S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2.$$

Suppose that $n \geq 4$. Find the value of c such that $\frac{c\bar{X}}{S^2}$ is an unbiased estimator of $\frac{\mu}{\sigma^2}$.

(20%) 5. Let $X_1, X_2, \ldots, X_n$ be a random sample from the population with cumulative distribution function

$$F(x) = \begin{cases} 0, & \text{if } x < 1; \\ \dfrac{1}{2} - \dfrac{\theta}{2}, & \text{if } 1 \leq x < 2; \\ 1 - \dfrac{\theta}{2}, & \text{if } 2 \leq x < 3; \\ 1, & \text{if } x \geq 3, \end{cases}$$

where $0 < \theta < 1$. Find the maximum likelihood estimator for θ.