國立中正大學
109 學年度碩士班招生考試
試 題
［第 1 節］

科目名稱	基礎數學
系所組別	數學系統計科學

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

(31%) 1. Evaluate the following limit and integral:

(6%) (a) $\lim_{n\to\infty}(\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\ldots+\frac{1}{\sqrt{n^2+n}})$

(6%) (b) $\lim_{x\to 0}(x^2+x+1)^{1/x}$

(6%) (c) $\int_0^2 xe^x dx$

(6%) (d) $\int_{-\infty}^{\infty} e^{-x^2/2}dx$

(7%) (e) $\int_0^1\int_{1-x}^2 xy^{1/2}dydx$

(10%) 2. Show that $e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}, \forall x\in \boldsymbol{R}$.

(10%) 3. Use method of Lagrange multipliers to find the minimun value of the function $f(x,y)=x^2+3y^2+2y$ subject to the constraints $x^2+y^2=1$.

(9%) 4. For an $n\times n$ square matrix A, write three statements that are equivalent to A is invertible.

(10%) 5. Let A be a real symmetric $n\times n$ matrix with eigenvalues $\lambda_1,\ldots,\lambda_n$ and corresponding orthonormal eigenvectors $e_1,\ldots,e_n$. If $\lambda_i>0; i=1,\ldots,n$, Show that A is also a positive definite matrix.

(10%) 6. Let e_1 and e_2 be eigenvectors of a symmetric matrix A that corresponding to distinct eigenvalues λ_1 and λ_2. Show that e_1 and e_2 are orthogonal.

(10%) 7. Let A=$\begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$. Find the eigenvalues and the corresponding eigenvectors of A^{100}.

(10%) 8. If y is an $n\times 1$ random vector with mean vector μ and covariance matrix Σ and if A is a symmetric matrix of constants, show that

$$E(y^tAy)=\mathrm{Tr}(A\Sigma)+\mu^tA\mu$$

where $E(\cdot)$ and $\mathrm{Tr}(\cdot)$ denote the expected value and the trace of a matrix, respectively.