國立中正大學
115學年度碩士班招生考試
試題

[第4節]

科目名稱	機率與統計
系所組別	數學系統計科學

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始40分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

國立中正大學 115 學年度碩士班招生考試試題

科目名稱：機率與統計　　　　　　　　　　　　本科目共 1 頁　第 1 頁

系所組別：數學系統計科學

1. (25%) Let X_i follow a Gamma distribution with parameters n_i and λ for $i = 1, 2$. That is, the probability density function of X_i is given by

$$f(x_i; n_i, \lambda) = \frac{\lambda}{\Gamma(n_i)}(\lambda X_i)^{n_i-1}\exp(-\lambda X_i),\ x_i > 0,\ i = 1, 2;$$

where $\Gamma(.)$ denotes the Gamma function. We assume that X_1 and X_2 are independent. Let $Y_1 = X_1/(X_1 + X_2)$ and $Y_2 = (X_1 + X_2)$. Please provide a detailed derivation and explanation for the following problems.

(a) (10%) Find the probability density function of Y_1

(b) (10%) Find the moment generating function of Y_2

(c) (5%) Use (b) to find the mean of Y_2.

2. (25%) Please verify your answer clearly.

(a) (15%) State and prove the Central Limit Theorem. (CLT).

(b) (10%) Suppose that $X_1, \cdots, X_n$ are independent and identically distributed random variables, and that $E(X_i^r) = \mu_r < \infty, r = 1,2,3,4$. Use the CLT to derive the limiting distribution of

$$Y_n = \frac{\sum_{i=1}^{n} X_i^2}{n}.$$

3. (28%) Let $X_1, \cdots, X_n$ be a random sample from the density

$$f(x_i; \theta) = \frac{2x}{\theta^2} I_{(0,\theta)}(x)$$

where $\theta > 0$ and $I_{(0,\theta)}(x)$ denotes the indicator function, which equals 1 if $0 < x < \theta$ and 0 otherwise. Please provide a detailed derivation and explanation for the following problems.

(a) (6%) Find the maximum likelihood estimator (M.L.E) of θ.

(b) (6%) Determine whether the M.L.E obtained in (a) is a sufficient statistic for θ.

(c) (8%) Determine whether the M.L.E obtained in (a) is a complete statistic for θ.

(d) (8%) Find the uniformly minimum variance unbiased estimator (U.M.V.U.E) of θ

4. (22%) Let $X_1, \cdots, X_n$ be a sample from a normal distribution with unknown mean μ and unknown variance σ^2. Please provide a detailed derivation and explanation for the following problems.

(a) (15%) Derive the likelihood ratio test of size α for testing $H_0: \mu = \mu_0$ against $H_1: \mu \neq \mu_0$ where μ_0 is a given number.

(b) (7%) Find a $100(1-\alpha)\%$ confidence interval of σ^2.