國立中正大學
113學年度碩士班招生考試
試 題
[第2節]

科目名稱	機率與統計
系所組別	數學系統計科學

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始40分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

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1. (15%) Let X_1 and X_2 be two independent Geometric random variables, each having a Geometric distribution with parameter P_i , i.e. the probability density function is

$$f(X_i, P_i) = P_i(1-P_i)^{X_i-1} \ , \ X_i = 1,2,\cdots\infty; \ i = 1,2$$

(a) (5%) Show that the Geometric distribution has the memoryless property.

(b) (10%) Let $Z = \min(X_1, X_2)$, find the probability density function (pdf) of Z.

2. (20%) Please verify your answer clearly.

(a) (10%) Use the Central Limit Theorem to find the limit as n approaches infinity of $2e^{-n}\sum_{k=0}^{n}\frac{n^k}{k!}$.

(b) (10%) Let $X_1,\cdots,X_n$ be independent and identically distributed random variables (i.i.d) from

Uniform$(0,\theta)$, $\theta > 0$. Let $\hat{\theta} = \frac{n+1}{n}\max(X_1,\cdots,X_n)$. Find the limiting distribution of $n(\hat{\theta}-\theta)$.

3. (25%) Suppose that $X_1,\cdots,X_n$ are i.i.d Bernoulli(p).

(a) (10%) Show that the variance of the maximum likelihood estimator of p attains the Cramer-Rao Lower Bound.

(b) (15%) For $n \geq 4$, Show that the product $X_1X_2X_3X_4$ is an unbiased estimator of p^4, and use this fact to find the U.M.V.U.E of p^4. (Hint: use Rao-Blackwell Lehmann-Scheffe Theorem)

4. (15%) Let $X_1,\cdots,X_n$ be i.i.d random variables from a Normal distribution with an unknown mean μ and a known variance σ^2. The hypotheses to be tested are

$$H_0: \mu = \mu_0 \ \text{ versus } \ H_1: \mu = \mu_1$$

where μ_0 and μ_1 are given constants, and $\mu_0 < \mu_1$. We consider the test:

Reject H_0 if $\bar{X} > c$, where c is a constant.

Determine values of c and the sample size n so the test satisfies the type I error probability is α and type II error probability is β. Please verify your answer clearly.

(Note that we used the notation z_α to denote the point having probability α to the right of it for a standard normal pdf.)

5. (25%) Let $X_1,\cdots,X_n$ and $Y_1,\cdots,Y_m$ be independent random samples from two normal distributions $N(\mu_1,\sigma^2)$ and $N(\mu_2,\sigma^2)$, respectively, where σ^2 is the common but unknown variance.

(a) (10%) Find the maximum likelihood estimator of $\log(\sigma^2)$

(b) (15%) Derive the likelihood ratio test of size α for testing $H_0: \mu_1 = \mu_2$ against $H_0: \mu_1 \neq \mu_2$