國立中正大學

108 學年度碩士班招生考試

試題

[第 1 節]

系所組別	數學系應用數學
科目名稱	微積分

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

1. (10 pts) Let $f(x) = \sin[\sec^{-1}(\ln x)]$ be defined on $[e, \infty)$. Find $(f^{-1})'(\frac{\sqrt{3}}{2})$.

2. (10 pts) Find the volume of the solid which is bounded by the cone $z = \sqrt{x^2+y^2}$ and the sphere $x^2+y^2+z^2 = 9$.

3. Evaluate the following limits

   a. (6 pts) $\displaystyle\lim_{x\to 0}\left(\frac{1}{\sin^2 x} - \frac{1}{x^2}\right)$.　　b. (6 pts) $\displaystyle\lim_{x\to\infty}(\ln x)^{\frac{1}{x}}$.

4. (12 pts) Find the absolute maximum and minimum of $f(x,y) = x^2+y^2+4x-4y$ on the region $x^2+y^2 \le 9$.

5. Let $f(x) = \begin{cases} x^4 \sin\left(\dfrac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$.

   a. (7 pts) Find $f''(0)$.

   b. (3 pts) Prove or disprove that f has an inflection point at $(0,0)$.

6. Evaluate the following integrals

   a. (6 pts) $\displaystyle\int \frac{\cos\theta}{1-\cos\theta}d\theta$.　　b. (6 pts) $\displaystyle\int_1^\infty \frac{1}{x\sqrt{x^2-1}}dx$.

7. (10 pts) Let C be the square cut from the first quadrant by the line $x = 1$ and $y = 1$. Evaluate the line integral $\displaystyle\oint_C xydy - y^2dx$.

8. (10 pts) A parabolic container (without top) is formed by revolving $y = x^2 - 1$ about the y-axis for $x \in [1,2]$, where x and y measured in centimeter. Suppose that a liquid is poured into the container at the rate of $2\text{cm}^3/\text{min}$, how fast is the level of the liquid rising when the depth of the liquid is 1cm.

9. a. (5 pts) Find the Maclaurin series for the function $f(x) = \sin^{-1} x$.

   b. (5 pts) Let $I_n = \displaystyle\int_0^{\frac{\pi}{2}} \sin^n x dx$. Show that $I_{2k+1} = \dfrac{(2^k k!)^2}{(2k+1)!}$.

   c. (4 pts) Use the results of **a.** and **b.** to show that $\displaystyle\sum_{n=1}^\infty \frac{1}{n^2} = \frac{\pi}{6}$.
   [Hint: Let $x = \sin^{-1}(\sin x)$ and take integral from 0 to $\frac{\pi}{2}$.]