國立中正大學105學年度碩士班招生考試試題

系所別：數學系應用數學　　科目：微積分

第1節　　

1. (10 points) Find the limits.

$$(a)\quad \lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{x}{x-1}\right) \qquad (b)\quad \lim_{x\to\infty}(x^3+1)^{1/x}$$

2. (10 points) (a) Assume that y is a differentiable function of x which satisfies the equation $x^2 + x\cos y = xy$. Use implicit differentiation to express $\frac{dy}{dx}$ in terms of x and y.

(b) Find an equation of the tangent line to the graph of the curve $x^2 + x\cos y = 2xy$ at the point (-1,0).

3. (10 points) Find the volume of the solid bounded above by the plane $2z = 4 + x$, below by the xy-plane, and the sides by the cylinder $x^2 + y^2 = 4$.

4. (10 points) Compute the integrals

$$(a)\quad \int x^{1/2}\ln x\, dx, \qquad (b)\quad \int \frac{x}{(x-1)(x^2+1)}\, dx$$

5. (10 points) (a) Prove that the series converges when $|x| < 1$.

$$\sum_{n=1}^{\infty}\frac{\ln n}{n}x^n$$

(b) Does the series converge at $x = 1$ and $x = -1$? Why?

6. (10 points) Define a sequence recursively by setting

$$a_1 = 1, \quad a_{n+1} = \sqrt{3a_n}, \quad n = 1, 2, 3, ...$$

(a) Show by induction that the sequence is bounded above.
(b) Show by induction that it is an increasing sequence.
(c) Find the limit $\lim_{n\to\infty} a_n$.

7. (10 points) Find the absolute maximum and absolute minimum of the function

$$f(x, y) = 4xy - x^2 - y^2 - 6x$$

on the triangular region bounded by the lines $y = 1$, $x = 0$ and $y = x$.

8. (10 points) (a) Sketch the region Ω that gives rise to the repeated integral

$$\int_0^1\int_{\sqrt{x}}^1 \sin\left(\frac{y^3+1}{2}\right)\, dy\, dx.$$

(b) Change the order of integration and evaluate the integral.

9. (10 points) Find the area of the region in the first quadrant bounded by the circle $x^2 + y^2 = 2$, the parabola $y = x^2$ and the x-axis.

10. (10 points) Let $f(x) = 2\cos x + x$ for $x \in [0, \pi]$.

(a) Find the intervals on which f is increasing and intervals on which f is decreasing.

(b) Find the intervals on which the graph of f is concave upward and intervals on which the graph is concave downward.

(c) Sketch the graph of f.