國立中正大學
109學年度碩士班招生考試
試題
[第1節]

科目名稱	微積分	
系所組別	數學系	
	數學系應用數學	

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始40分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

Part I. Multiple choice problem. (5 points each. 10 points in total.)

1. (5 points) 單選 $f(x)$ is a continuous function on [0,1], which one of following condition can ***not*** imply $f(x) = 0\ \forall x \in [0,1]$.

   (a) $\int_0^1 |f(x)|dx = 0$.　　(b) $\int_0^1 f(x)dx = 0$.　　(c) $\int_0^x f(t)dt = 0\ \forall x \in [0,1]$.

2. (5 points) 單選 $a_n, b_n > 0\ \forall n \geq 1$. Both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent, which one of following is correct?

   (a) $\sum_{n=1}^{\infty} \sqrt{a_n}$ is convergent.　(b) $\lim_{n\to\infty} \sqrt{(a_n)^2 + (b_n)^2} > 0$.

   (c) $\sum_{n=1}^{\infty} (a_n + b_n)x^n$ is convergent on [-1,1].

Part II. Short answer problem. (Problem 3-10, 5 points each; Problem 11-16, 6 points each. 76 points in total.)

Simply write your answers on the answer sheet.

3. (5 points) Write the precise definition of $\lim_{x\to 1} f(x) = \infty$.

4. (5 points) Find a value of c that makes the function $f(x) = \begin{cases} \frac{2x^2-(3x+1)\sqrt{x}+2}{x-1} & ,x \neq 1 \\ c & ,x = 1 \end{cases}$ continuous at x=1.

5. (5 points) Find $\frac{d}{dx}(x^x)$ for $x > 0$. (Hint: $x^x = e^{x \cdot \ln(x)}$.)

6. (5 points) Evaluate $\int \frac{1}{(x^2-1)^2} dx$.

   (Hint: $\frac{1}{(x^2-1)^2} = \frac{ax+b}{(x+1)^2} + \frac{cx+d}{(x-1)^2} = \frac{a}{(x+1)} + \frac{b-a}{(x+1)^2} + \frac{c}{(x-1)} + \frac{c+d}{(x-1)^2}$.)

7. (5 points) f is a continuous function on real line and $\int_1^{2x} f(t)dt = x \cdot \sin(\pi x)$. Find $f(1)$.

   (Hint: take derivative on both sides.)

8. (5 points) Find the interval of convergence for $\sum_{n=1}^{\infty}\frac{n^n}{n!}(x+3)^n$.

(Hint: $\lim_{t\to 0}(1+\mathrm{t})^{1/\mathrm{t}}=e$.)

9. (5 points) Evaluate the arc length of the curve $r=1-\cos(\theta)$ on $0\le\theta\le\pi$.

(Hint: $\mathrm{L}=\int_{\alpha}^{\beta}\sqrt{r^2+(\frac{dr}{d\theta})^2 d\theta}$.)

10. (5 points) Find the curvature κ for the curve $\mathbf{r}(t)=t\mathbf{i}+(\ln(\cos(t)))\mathbf{j}$, $-\pi/2<t<\pi/2$.

(Hint: Find unit tangent vector **T** first, then $\kappa=\left|\frac{d\mathbf{T}}{ds}\right|=\frac{1}{|\mathbf{v}|}\left|\frac{d\mathbf{T}}{dt}\right|$.)

11. (6 points) Evaluate $\int_0^1\int_x^1 y^2e^{xy}\ dydx$ by reversing the integration order.

12. (6 points) $f(x,y)=2x+4y-x^2-y^2$. D is the intersection region of $x\ge 0$, $y\ge 0$, and $y\le 9-x$.

The global maximum value of $f(x,y)$ on D is M; the global minimum value of $f(x,y)$ on D is m. What is $M+m$?

13. (6 points) Find the point(s) on the surface $z=xy+1$ closest to the origin point (0,0,0).
(Hint: minimize $x^2+y^2+z^2$.)

14. (6 points) Find the volume of solid bounded below by the hemisphere $\rho=1$, $z\ge 0$ and above by the cardioid of revolution $\rho=1+\cos\Phi$.
(Hint: use spherical coordinates)

15. (6 points) Evaluate the integral $\oint_C x(y^2)dy-y^3dx$, where C is the square cut from the first quadrant by the lines $x=0$, $x=1$, $y=0$ and $y=1$. (Hint: use Green's Theorem)

16. (6 points) Find the outward flux of the vector field $\mathbf{F}=(y-x)\mathbf{i}+(z-y)\mathbf{j}+(x-y)\mathbf{k}$ across the boundary of the region D, where D is the cube bounded by the planes $x=\pm1$, $y=\pm1$, and $z=\pm1$.

Part III. Writing problem (14 points).

Write all your work on the answer sheet.

17. $g(x,y)=\begin{cases}\frac{x^2y}{x^4+y^2} & ,(x,y)\neq(0,0)\\ 0 & ,(x,y)=(0,0)\end{cases}$.

a) (5 points) Find $\lim\limits_{(x,y)\to(0,0)} g(x,y)$. (if the limit exist, evaluate the limit; if not, show it does not exist.)

b) (5 points) Find $\frac{\partial g}{\partial x}(0,0)$. (if the partial derivative exists, evaluate it; if not, show it does not exist.)

c) (4 points) Show g is not differentiable at (0,0).