1. Find

(i) (7pts) $\dfrac{d}{dx}\displaystyle\int_{\cos x}^{x^3} \sin(t^2)\, dt$, (ii) (7pts) $\dfrac{d}{dx}(1-\sin x)^{1/x}$, and (iii) (6pts) $\displaystyle\lim_{x\to 0^+}(1+\sin x)^{-1/x}$.

2. Evaluate

(i) (10pts) $\displaystyle\int e^{3x}\sin 2x\, dx$, and (ii) (10pts) $\displaystyle\int \frac{dx}{(x^2+4x+5)^{3/2}}$.

3. (a) (10pts) Test for convergence $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n}}{n+1}$.

(b) Find (i) (7pts) the Taylor series centered at $a=1$ for the function $f(x)=\ln x$, and

(ii) (3pts) its interval of convergence?

4. (a) (10pts) Find the absolute extrema for $f(x,y)=6x^2-8x+2y^2-3$ over the region $\mathcal{R}$ enclosed by $x^2+y^2=1$.

(b) (10pts) Use polar coordinates to evaluate

$$\int_0^3\int_x^{\sqrt{18-x^2}}\frac{1}{x^2+y^2+1}\,dy\,dx.$$

5. (a) (6 pts) Evaluate

$$\oint_\gamma (e^x+3y)\,dx+(2x-e^{y^2})\,dy,$$

where γ is the ellipse $\dfrac{x^2}{4}+\dfrac{y^2}{9}=1$.

(b) (14 pts) Let $\mathcal{R}$ be the region enclosed by the upper hemisphere $x^2+y^2+z^2=9$, $0\le z\le 3$, and the plane $z=0$. Justify the divergence theorem for $\mathbf{F}=x\mathbf{i}+y\mathbf{j}+(z-1)\mathbf{k}$.