(20%)1. Suppose f is a real-valued and continuous function defined on $[a, b]$. Show that $R(f) = \{f(x) \mid x \in [a, b]\}$ is a closed interval.

(15%) 2. Let $A \subseteq \mathbf{R}^n$, and $\bar{A}$ be the closure of A. Show that

(a) $\bar{A}$ is a closed set of $\mathbf{R}^n$, and

(b) $\bar{\bar{A}} = \bar{A}$.

(20%) 3. Suppose f is an increasing function defined in (a, b). Show that f in continuous in (a, b), except possibly countable many points.

(25%) 4. Suppose f is real-valued and continuous in $[0, \infty)$, and the improper integral $\int_0^\infty |f(x)|dx$ exists. S etthe Fourier transform of f by

$$\hat{f}(\xi) = \int_0^\infty f(x)\cos(x\xi)dx, \quad \xi \in [0, \infty).$$

Show that

(a) $\hat{f}$ is a function defined in $[0, \infty)$,

(b) $\hat{f}$ is a continuous function defined in $[0, \infty)$, and

(c) $\lim_{\xi \to \infty} \hat{f}(\xi) = 0$.

(20%) 5. Let $I = \prod_{i=i}^{n} [a_i, b_i]$ and f be a real-valued and continuous function defined on I. Show that f is Riemann integrable on I.