國立中正大學
109學年度碩士班招生考試
試題
[第2節]

科目名稱	線性代數
系所組別	數學系

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始40分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

Attentions!! In the following problems, $\mathbb{R}$ denotes the field of real numbers, and $\mathbb{Z}_2 = \{0, 1\}$ denotes the prime field of characteristic 2.

(1) Let V be an $\mathbb{R}$-vector space. Let U and W be subspaces of V. Show that $U \cup W$ is a subspace of V if and only if $U \subseteq W$ or $W \subseteq U$. (15 points) Is this assertion still true when V is a $\mathbb{Z}_2$-vector space instead? (5 points)

(2) Let $\mathcal{P}_4$ be the $\mathbb{R}$-vector space of polynomials of degree ≤ 4 which have coefficients in $\mathbb{R}$.

(a) Find the dimension of

$$W = \mathrm{Span}\,(1 + x^3,\ 1 + x + x^2,\ 1 + x + x^3).$$

(10 points)

(b) Suppose U is a subspace of $\mathcal{P}_4$ such that $U + W = \mathcal{P}_4$. What is the least possible dimension for U? (10 points)

(3) Let A be a square matrix with entries in $\mathbb{R}$. Suppose the characteristic polynomial of A is $(\lambda - 1)^3(\lambda - 2)^3$. If in addition we know that

$$A^4 - A^3 - 7A^2 + 13A - 6I = 0 \quad \text{and} \quad A^2 - 3A + 2 \neq 0.$$

Give the Jordan form of A. (20 points)

(4) Let V be a complex inner product space with the inner product $\langle -, - \rangle$. Show that

$$\|v + w\|^2 + \|v - w\|^2 = 2\|v\|^2 + 2\|w\|^2.$$

(15 points)

(5) Suppose A is a square matrix with entries in $\mathbb{R}$ such that A^2 has a positive eigenvalue. Show that A has at least one real eigenvalue. (25 points)