國立中正大學110學年度碩士班招生考試試題

科目名稱：線性代數　　　　　　　　　　　　　　　本科目共 1 頁　第 1 頁
系所組別：數學系

INSTRUCTION: To earn partial credits, show your work. *No calculators allowed*.

1. (25pts) Let $\mathbf{A}$ be the matrix:

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

   (i) (7pts) Find the characteristic polynomial of $\mathbf{A}$.

   (ii) (8pts) Find the eigenspaces.

   (iii) (10pts) Find the minimal polynomial of $\mathbf{A}$.

2. (25pts) Let $\mathbf{B}$ be the matrix:

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 5 \\ 3 & 2 & 1 \end{pmatrix}$$

   (i) (13pts) *Justify* Cayley-Hamilton theorem for the matrix $\mathbf{B}$.

   (ii) (12pts) Using (i), give an expression of the inverse $\mathbf{B}^{-1}$ in terms of $\mathbf{I}_3, \mathbf{B}, \ldots$, and find the second column vector of $\mathbf{B}^{-1}$.

3. (25pts) Let $\beta_1 = \{t-1, t^2+1, t^2-t\}$ and $\beta_2 = \{2t^2-t-3, 5t^2-2t-3, -2t^2+5t+5\}$ be a pair of ordered bases for $\mathbf{P}_2(\mathbb{R})$, all real polynomials with a degree $n \leq 2$.

   Find the change of coordinate matrix that change β_2-coordinates into β_1-coordinates.

4. (25pts) Let $\mathbf{V} = \mathbf{P}(\mathbb{R})$, all real polynomials, with the inner product $\langle p, q \rangle = \int_{-1}^{1} p(t)q(t)dt$.

   (i) (15pts) For the subspace $\mathbf{P}_2(\mathbb{R})$, apply the Gram-Schmit process to the standard ordered basis $\beta = \{1, t, t^2\}$ to obtain an orthonormal basis γ for $\mathbf{P}_2(\mathbb{R})$.

   (ii) (10pts) Find the orthogonal projection of t^3 on $\mathbf{P}_2(\mathbb{R})$.