國立中正大學
114 學年度碩士班招生考試
試 題

[第 3 節]

科目名稱	微積分
系所組別	數學系
	數學系應用數學

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

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1. (12 pts) Let $f(x) = \begin{cases} \dfrac{\sin x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ and define $\mathrm{Si}(x) = \displaystyle\int_0^x f(t)dt$. Evaluate the limit $\lim_{x\to 0} \dfrac{\mathrm{Si}(x) - x}{x^3}$.

Hint: Use Fundamental Theorem of Calculus $\dfrac{d}{dx}\displaystyle\int_0^x g(t)dt = g(x)$.

2. (12 pts) Find the equations of tangent lines to the graph of $f(x) = x^3 - x$ that passes through the point $(-2, 2)$.

3. (12 pts) Evaluate the following integrals

a. (6 pts) $\displaystyle\int_0^{\frac{\pi}{2}} \sin^4 x dx$.　　**b.** (6 pts) $\displaystyle\int_2^{\infty} \frac{dx}{x^2\sqrt{4x+1}}$.

4. (12 pts) Let $p \in \mathbb{R}$. Determine values of p such that $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n^p}$ converges in three cases **a.** (4 pts) $p \in (-\infty, 0]$; **b.** (4 pts) $p \in (0, 1]$; **c.** (4 pts) $p \in (1, \infty)$.

5. (14 pts) Let $f(x) = \dfrac{1}{\sqrt{1+x}}$.

a. (4 pts) Let $k \in \mathbb{N}$, find $f^{(k)}(x)$.

b. (5 pts) Find the Maclaurin series (Taylor series centered at 0) of $f(x)$.

c. (5 pts) Determine the radius of convergence to the Maclaurin series.

6. (12 pts) Evaluate the integral

$$\int_{\frac{1}{\sqrt{2}}}^{1}\int_{\sqrt{1-x^2}}^{x} xydydx + \int_{1}^{\sqrt{2}}\int_{0}^{x} xydydx + \int_{\sqrt{2}}^{2}\int_{0}^{\sqrt{4-x^2}} xydydx.$$

7. (12 pts) Is the vector field $\mathbf{F}(x,y) = (x+\sin y)\mathbf{i} + (x\cos y - 2y)\mathbf{j}$ conservative? If so, find a potential function f such that $\nabla f = \mathbf{F}$.

8. (14 pts) Evaluate the line integral $\int_C (3+e^{x^2})dx + (\tan^{-1} y + 3x^2)dy$, where C is the boundary of the region between the circles $x^2+y^2=1$ and $x^2+y^2=4$ in the first quadrant.
Hint: You can use Green's Theorem.