國立中正大學
112學年度碩士班招生考試
試 題
[第1節]

科目名稱	微積分
系所組別	數學系

—作答注意事項—

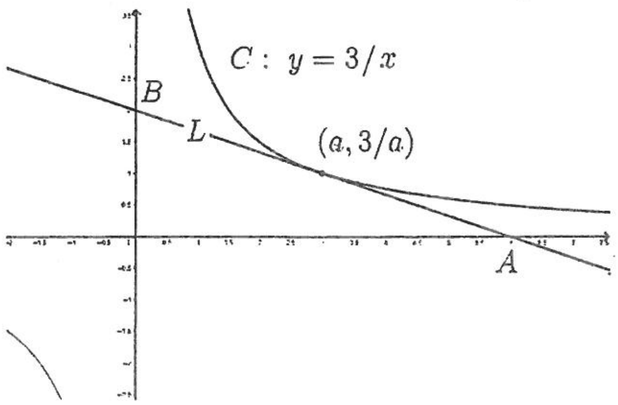
※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始40分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

1. (10 points) Find $\lim_{x \to 0^+} x^x$.

2. Let $f(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0; \\ 0, & x = 0. \end{cases}$
   (a) (5 points) Find $\lim_{x \to 0} f(x)$.
   (b) (3 points) Is $f(x)$ continuous at $x = 0$?
   (c) (6 points) Find $f'(x)$ for $x \neq 0$.
   (d) (6 points) Find $f'(0)$.
   (e) (3 points) Is $f(x)$ differentiable at $x = 0$?

3. Let C be the curve given by $y = 3/x$ and let L be the tangent line to the curve C at the point (a, b) where $a > 0$. Let A be the intersection of L and the x-axis and B be the intersection of L and the y-axis.

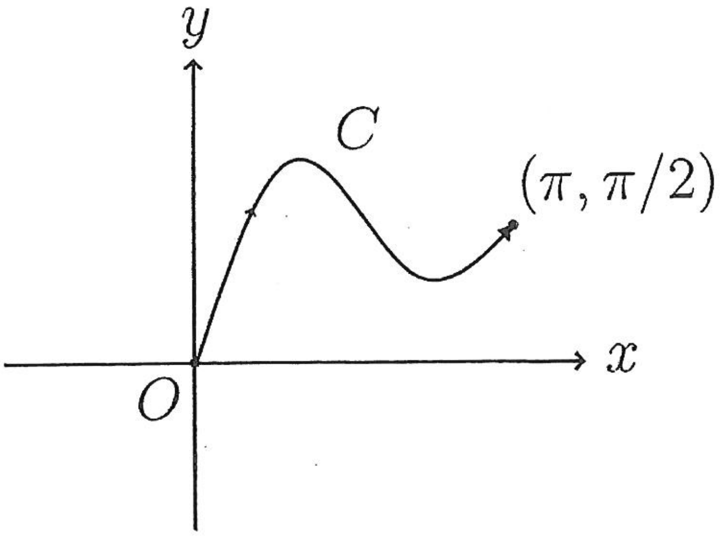


   (a) (6 points) Find the equation for L.
   (b) (5 points) Show that $A = (2a, 0)$ and $B = (0, 6/a)$.
   (c) (10 points) Find the shortest possible length of $\overline{AB}$ among $a > 0$.

4. (a) (5 points) Give the Maclaurin series (the Taylor series at $x = 0$) of $\dfrac{1}{1+x^2}$.
   (b) (3 points) Find the radius of convergence of the Maclaurin series in (a).
   (c) (5 points) Give the Maclaurin series of $\displaystyle\int_0^x \frac{dt}{1+t^2}$.
   (d) (3 points) Find the radius of convergence of the Maclaurin series in (b).
   (e) (8 points) Show that

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)(\sqrt{3})^{2n+1}}$$
$$= \frac{1}{\sqrt{3}} - \frac{1}{3(\sqrt{3})^3} + \frac{1}{5(\sqrt{3})^5} - \frac{1}{7(\sqrt{3})^7} + - \cdots = \frac{\pi}{6}.$$

5. Let $\mathbf{F}(x) = (2x\cos y + y\cos x,\ -x^2\sin y + \sin x + 1)$.
   (a) (6 points) Is the vector field $\mathbf{F}(x)$ conservative? If yes, find its potential function.
   (b) (6 points) Find $\int_C \mathbf{F} \cdot d\mathrm{r}$ where C is a curve from the origin O to $(\pi, \pi/2)$.



6. (10 points) Evaluate $\int_0^1 \int_y^1 \cos(x^2+1)\,dx\,dy$.